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Einstein-Podolsky-Rosen- (EPR) and the more powerful Mayers-Lo-Chau attack impose a serious constraint on quantum bit commitment (QBC). As a way to circumvent them, it is proposed that the quantum system encoding the commitment chosen by the committer (Alice) should be initially prepared in a separable quantum state known to and furnished by the acceptor (Bob), rather than Alice. Classical communication is used to conclude the commitment phase and bind Alice's subsequent unveiling. Such a class of secure protocols can be built upon currently proposed QBC schemes impervious to a simple EPR attack. A specific scheme based on the Brassard-Cr peau-Josza-Langlois protocol is presented here as an example.

I. INTRODUCTION

Quantum cryptography is concerned with harnessing the principles of quantum mechanics to create secure cryptosystems [1,2]. It can broadly be divided into quantum key distribution (QKD) [2,3], which is concerned with secure sharing of cryptographic keys, and a host of other schemes, such as quantum coin tossing [4–6], quantum oblivious mutual identification [7], quantum oblivious transfer [8] and “two party secure computation” (TPSC) [9], essentially concerned with secure processing of private information to reach a public decision. The latter schemes depend on the validity of quantum bit commitment (QBC) [2,5], a quantum cryptographic primitive for secure information processing. In a concrete realization of bit commitment, Alice writes 0 or 1 on a note, puts it in a safe, which she hands over to Bob. Upon Bob choosing to enter the transaction, she gives him the key to the safe. The main point is that Alice cannot cheat by changing her mind after handing Bob the safe, nor can Bob cheat by finding out about Alice's decision unless she gives him the key. A secure bit commitment is one which is (at least, exponentially) binding on Alice *and* unconditionally concealing (of her commitment) from Bob and thus prevents either party from cheating.

That entanglement can undermine QBC was first realized by Bennett and Brassard [2], who pointed out that the BB84 scheme [2] was insecure against an Einstein-Podolsky-Rosen (EPR) attack by Alice [10], i.e., a deception where she sends Bob part of entangled photons instead of ones in a definite polarization state, and waits until after the initial phase of the protocol to measure the part she retains. A subsequent proposition, namely the BCJL protocol [4,5], though impervious to an EPR attack, is nevertheless rendered insecure by an entanglement-based attack independently uncovered by Lo and Chau [11,12] and Mayers [13]. The essence of their proof is that if the protocol is secure against Bob, Alice can cheat by supplying him a pure state entangled system, and switching her commitment by local unitary operations. It has come to be accepted that QBC simultaneously secure against both Alice and Bob is impossible, though a trade-off is permitted [14].

Almost all QBC schemes we know conform to a model wherein the quantum system encoding the committer's (Alice's) commitment b is prepared by her. She is the sender. The party that accepts her commitment, the “acceptor” called Bob, is the receiver of the quantum information. The present article explores whether QBC can be made secure if Bob, rather than Alice, prepares and sends the initial state of the encoding quantum system, and Alice is the receiver who “inscribes” her commitment on the state furnished by Bob. The motivation to do so is quite straightforward, given that the insecurity of QBC so far stems from Alice preparing the initial state as she likes.

II. THE MODIFIED BCJL SCHEME

In a typical QBC scheme, Alice prepares a quantum state consisting of a pre-agreed number n of photons in a pure product state determined by her commitment b ($\in \{0, 1\}$). Four possible preparations of a photon are permitted: in the rectilinear basis (denoted $+$), with horizontal (denoted 0) or vertical (denoted 1) polarization; else, in the diagonal

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basis (denoted \times) with polarization oriented at 45° (denoted 0) or 135° (denoted 1). She sends these photons to Bob as a piece of evidence of her commitment. If Bob signals his acceptance to enter the transaction, she unveils b and the preparation information of her photons, which Bob then verifies by measuring their polarizations. In order that Bob should not cheat, the density matrices of the system given to Bob corresponding to $b = 0, 1$ – namely ρ_0^B, ρ_1^B – should be (almost) indistinguishable. On the other hand, the preparation of the photons she sends Bob should be binding on her. As noted earlier, if a Mayers-Lo-Chau (MLC) attack can be launched, then these two requirements are mutually exclusive [11–13].

A simple way to circumvent an EPR or MLC attack is to prevent Alice from launching one. The main point of the present article is that Bob can achieve this by preparing a separable n -particle quantum state and sending it to Alice to inscribe her commitment. We need to show that any subsequent action by Alice will not permit her to launch an attack. Then, a classical communication from Alice is sufficient to guard against her cheating. The hallmarks of the class of protocols we envisage are (1) a reverse quantum communication, wherein Alice and Bob have exchanged their traditional sender-receiver roles, and (2) a classical communication from Alice to signal the end of the commitment phase.

We present a version of the BCJL protocol modified to include these two features. The proof of its security is given thereafter. The modified BCJL scheme consists of two phases, the commitment phase and unveiling phase, enumerated below. An intervening phase of arbitrary duration, referred to as the holding phase, is implicit but ignored in the analysis. Since we only wish to present a proof of principle, discussion on error correction is not included here.

1. Commitment phase:

- (a) Alice and Bob agree upon an n -bit code \mathcal{C} (with some required properties).
- (b) They also agree upon a random n -bit string $r \in \{0, 1\}^n$.
- (c) Bob chooses a random n -bit string $R_B \in \mathcal{C}$.
- (d) He chooses a random n -bit string $\eta \in \{+, \times\}^n$ and prepares the state $|R_B\rangle_\eta = |R_B(1)\rangle_{\eta(1)} \otimes \cdots \otimes |R_B(n)\rangle_{\eta(n)}$ and sends it to Alice.
- (e) She chooses a random n bit string $\theta \in \{+, \times\}^n$ and measures Bob's photons in bases θ , obtaining outcomes $R_A \in \{0, 1\}^n$.
- (f) To encode her commitment b , Alice checks whether $\{R_A \in \mathcal{C} \mid r \odot R_A = b\}$, where the symbol \odot denotes scalar product modulo 2, or the parity of the bitwise AND operation. If the check succeeds (fails), she excludes a photon at a randomly chosen position x where she obtained outcome 0 (1), to obtain R'_A such that $\{R'_A \in \mathcal{C} \mid r \odot R'_A = b\}$.
- (g) She communicates to Bob the value of x . This announcement serves as a piece of evidence of her commitment.

2. Unveiling phase:

- (a) Alice announces b , R_B and θ .
- (b) Bob verifies that: $r \odot R'_B = b$.
- (c) He also verifies that whenever $\eta(i) = \theta(i)$, $R_A(i) = R_B(i)$.

Even though Bob prepares and thus knows the state $|R_B\rangle_\eta$ he sent Alice, without access to her outcomes where $\eta(i) \neq \theta(i)$, he cannot deduce b . In fact, he doesn't even know where η and θ don't match. Further, Alice need not fear that by sending in an entangled state rather than a separable state $|R_B\rangle_\eta$, Bob can hope to get information about her measurement outcomes. The very nature of quantum measurement (assumed to be a von Neumann projection or, more generally, a positive operator valued measure) prohibits Bob from knowing anything about her action, because Bob's local density matrix is unaffected by Alice's measurement. Therefore, even empowered with a quantum computer, Bob cannot hope to deduce her outcomes by observations local to him. One way to view this is that if this were not the case, then Alice could transmit superluminal signals to Bob according to her choice of θ [15]. Another deterrent for Bob, as shown below, is that Alice could launch an MLC attack against him if the system remains entangled with a hidden system on his side at the time of her measurement. Therefore the modified protocol is indeed secure against Bob.

Since Bob knows the exact pure, separable state he sent her, any entanglement-based attack cannot be launched by Alice. Moreover the no-cloning theorem [16] prevents her from knowing the exact state Bob sent her. Were this not so, she could find out the exact state Bob sent her and cheat by unveiling some $\theta(i)$ in the wrong basis and claim any

outcome she likes. Thus, if she decides to switch her commitment in the unveiling phase, the best she can do is to flip some $R'_B(i)$ with outcome 1, and announce the dishonest R'_B to Bob. But her probability for getting away without Bob noticing is $1/2$. Therefore, if the scheme is implemented on s rounds, her probability for cheating successfully falls exponentially as $(\frac{1}{2})^s$. Alice's classical communication in step 1(g) serves the crucial dual role of signaling Bob that the commitment phase is over, as well as binding Alice's to-be-unveiled commitment. This step is necessary because he cannot deduce based on local measurements whether she has measured or not. On the other, a mere intimation on her part is clearly insufficient, since she could simply lie that she measured, while actually intending to delay the decision on her commitment. It must bind her future unveiling, but without betraying b . Therefore, we expect that any proposed scheme that constrains outcomes of measurements— with the result that ρ_0^B and ρ_1^B slightly differ (eg., the BCJL scheme)— rather than measurement basis (eg., the BB84 scheme), permits a secure version along the lines given above.

III. ENTANGLEMENT WEAKENS SECURITY AGAINST ALICE

If Bob were to send Alice half of EPR pairs instead of the pure state $|R_B\rangle_\eta$, the above protocol would still be secure against EPR attacks by Alice. However, Alice can launch an MLC attack. To this end, she does not execute the measurement in 1(e). For step 1(g), Alice chooses a random photon for exclusion. Suppose each of Alice's remaining photons is half of an EPR pair in the state $|\phi\rangle = \frac{1}{\sqrt{2}}(|0\rangle_+|0\rangle_+ + |1\rangle_+|1\rangle_+)$, the other half being sent to Bob. The $2(n-1)$ particle state is given by

$$|\Phi\rangle_{AB} = 2^{-(n-1)/2} \sum_{j=1}^{2^{n-1}} |j\rangle_A \otimes |j\rangle_B \quad (1)$$

in the $+$ basis. The register A is sent to Alice, while B remains with Bob. Since the BCJL permits $N \equiv 2^{n-1} \cdot 2^{n-2}$ states encoding a given commitment, she augments register A she received from Bob by adding ancilla C , so that $\mathcal{H}_C \otimes \mathcal{H}_A$, the Hilbert space of the combined CA system, has the dimension N [17].

$$|\Psi\rangle_{CAB} = 2^{-(n-1)/2} \sum_{j=1}^{2^{n-1}} |e_j\rangle_C \otimes |j\rangle_A \otimes |j\rangle_B \equiv 2^{-(n-1)/2} \sum_{j=1}^N |f_j\rangle_{CA} \otimes |j\rangle_B, \quad (2)$$

where, for $j > 2^{n-1}$, we set $|j\rangle_B = 0$, or null state, but retain the $|j\rangle_B$'s as before for $j \leq 2^{n-1}$.

We denote an ensemble of states in the BCJL scheme encoding commitment $b = 0$ by $\{\sqrt{p_j}|0_j\rangle\}$ where $\sum_j p_j |0_j\rangle\langle 0_j| \equiv \rho_0^B \approx \text{Tr}_A(|\Phi\rangle_{AB}\langle\Phi|_{AB})$. To produce it, she determines the $N \times N$ unitary mixing matrix U^0 such that:

$$2^{-(n-1)/2} |j\rangle_B = \sum_{k=1}^N U_{jk}^0 \sqrt{p_j} |0_k\rangle_B. \quad (3)$$

Denoting $|g_j\rangle_{CA} \equiv \sum_{k=1}^N U_{jk}^0 |f_k\rangle_{CA}$, we rewrite Eq. (2) as:

$$|\Psi\rangle_{CAB} = \sum_{j=1}^N \sqrt{p_j} |g_j\rangle_{CA} \otimes |0_j\rangle_B. \quad (4)$$

By measuring in the $\{|g_j\rangle\}$ basis of the AC system, she produces a state compatible with $b = 0$. On the other hand, to convince Bob she is committed to $b = 1$, starting from Eq. (4), Alice applies a unitary transformation to the CA register to produce an ensemble $\{\sqrt{q_j}|1_j\rangle_B\}$ where $\sum_j q_j |1_j\rangle\langle 1_j| \equiv \rho_1^B \approx \text{Tr}_A(|\Phi\rangle_{AB}\langle\Phi|_{AB})$ on Bob's side. Such a transformation exists since $\rho_0^B \approx \rho_1^B$ [11–13]. Alternatively, she can directly determine the $N \times N$ unitary mixing matrix U^1 of her incremented system needed to generate $\{\sqrt{q_j}|1_j\rangle_B\}$ state.

The attack works if Alice knows the exact entangling state $|\phi\rangle$, in this case) Bob used. Nevertheless, it is interesting that the danger of an MLC attack exists even when Bob prepares the state that is to encode Alice's commitment. The main lesson is that the use of entanglement in any form undermines the security of QBC against Alice.

The vicissitudes of QBC's fate are a bit remarkable and indicate its subtle nature. Are there simple fundamental causes why the present version of QBC succeeds? In answer, one might say, as in QKD: Heisenberg uncertainty, quantum no-cloning and "causality"—that Bob cannot deduce Alice's action without classical input from her. It is interesting that essentially the same properties of quantum information that guarantee security between collaborating parties in quantum key distribution [2] do the same even when the two parties are mutually distrustful.

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